

CERN-TH.7264/94
hep-ph/9405353

Weak Decays of Heavy–Quark Systems

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Abstract

The recent theoretical progress in the description of semileptonic decays in the framework of the heavy mass expansion is summarized. Both inclusive and exclusive decays are considered.

Contribution to the XXIXth Rencontres de Moriond,
Electroweak Interactions and Unified Theories,
Méribel, France, March 12–19, 1994.

CERN-TH.7264/94
May 1994

1 Introduction

Hadrons containing heavy quarks have attracted a lot of theoretical attention recently, since there has been an improvement in the understanding of heavy hadrons. The Standard Model describes the decays of the heavy quarks, but due to the confinement property of the strong interactions only the decays of heavy hadrons may be observed. In order to make contact between the quark and the hadron level, one has to deal with the QCD bound-state problem, which has not yet been solved. For this reason one has to use models to describe the weak decays of hadrons.

However, it has been observed recently by various authors [1, 2] that the model dependence may be reduced substantially for b and c quarks. The mass of these quarks is much larger than the energy scale determined by the light degrees of freedom, and it is convenient to switch to an effective theory description in which the dynamical degrees of freedom of the heavy quark are “integrated out”; in other words, the heavy quark becomes an infinitely heavy, static source of a colour field, moving with a fixed velocity.

This “Heavy Quark Effective Theory” (HQET) corresponds to a systematic expansion of the full QCD Green-functions in inverse powers of the heavy-quark mass(es). The leading terms of this heavy mass expansion may be constructed from a renormalizable Lagrangian, which may be obtained from QCD by following the usual steps to construct an effective theory [3].

In the heavy mass limit, two new symmetries appear, which are not present in QCD. The first symmetry is the heavy flavour symmetry, which is due to the fact that the interaction of the quarks with the gluons is flavour blind and in the heavy mass limit all heavy quarks act as a static source of colour. Formally this corresponds to an $SU(2)$ symmetry relating b into c quarks moving with the same velocity. The second symmetry is the spin symmetry of the heavy quark. The interaction of the heavy quark spin with the “chromomagnetic” field is inversely proportional to the heavy mass and hence vanishes in the infinite mass limit. As a consequence, the rotations for the heavy quark spin become an $SU(2)$ symmetry, which holds for a fixed velocity of the heavy quark.

Corrections to the limit $1/m_Q = 0$ may be studied systematically in the framework of HQET. The corrections are given as power series expansions in two small parameters. The first small parameter is the strong coupling constant taken at the scale of the heavy quark $\alpha_s(m_Q)$. This type

of correction may be calculated systematically using perturbation theory in HQET. The second type of correction is characterized by the small parameter $\bar{\Lambda}/m_Q$, where $\bar{\Lambda}$ is a scale of the light QCD degrees of freedom, e.g. $\bar{\Lambda} \sim m_{hadron} - m_Q$. In the effective theory approach this type of correction enter through operators of higher dimension, the matrix elements of which have to be parametrized in general by additional form factors.

Inclusive decays may be treated in the heavy mass expansion by performing an operator product expansion much as is done in deep inelastic scattering [4]. The leading term in this expansion turns out to be the decay of a free quark, and corrections may be parametrized by forward matrix elements of higher dimensional operators.

In this contribution I shall focus on semileptonic decays of heavy mesons and consider in the next section exclusive heavy-to-heavy transitions, with some emphasis on the model-independent determination of V_{cb} . In section 3 we summarize the recent developments in the application of the heavy mass expansion to inclusive semileptonic rates and decay spectra.

2 Exclusive Semileptonic Decays

The heavy-quark spin flavour symmetry strongly reduces the number of form factors in exclusive heavy-to-heavy transitions [1]; in fact, for mesonic heavy-to-heavy transitions there is only one form factor, called the Isgur–Wise function. For a transition between heavy ground state mesons \mathcal{H} (either pseudoscalar or vector) with heavy flavour f (f') moving with velocities v (v'), one obtains in the heavy-quark limit

$$\langle \mathcal{H}^{(f')}(v') | \bar{h}_v^{(f')} \Gamma h_v^{(f)} | \mathcal{H}^{(f)}(v) \rangle = \xi(vv') C_\Gamma(v, v'). \quad (1)$$

Here $h_v^{(f)}$ is the field operator annihilating a heavy quark of flavour f , moving with velocity v ; Γ is some arbitrary Dirac matrix. The Isgur–Wise function $\xi(vv')$ contains all the non-perturbative information for the heavy-to-heavy transition, while the coefficient $C_\Gamma(v, v')$ may be calculated from the symmetries. Furthermore, heavy-quark symmetry fixes the value of ξ at the point $v = v'$ to be

$$\xi(vv' = 1) = 1, \quad (2)$$

since the current $\bar{h}_v^{(f')} \Gamma h_v^{(f)}$ is one of the generators of heavy-flavour symmetry.

In the following we shall treat the b and the c quarks as heavy. Then (1) implies that the decays $B \rightarrow D\ell\bar{\nu}_\ell$ and $B \rightarrow D^*\ell\bar{\nu}_\ell$ are described by a single form factor, the Isgur–Wise function. In general, the relevant matrix elements are given in terms of six form factors

$$\begin{aligned}\langle D(v')|\bar{c}\gamma_\mu b|B(v)\rangle &= \sqrt{m_B m_D} [\xi_+(y)(v_\mu + v'_\mu) + \xi_-(y)(v_\mu - v'_\mu)] \quad (3) \\ \langle D^*(v', \epsilon)|\bar{c}\gamma_\mu b|B(v)\rangle &= i\sqrt{m_B m_{D^*}} \xi_V(y) \varepsilon_{\mu\alpha\beta\rho} \epsilon^{*\alpha} v'^\beta v^\rho \\ \langle D^*(v', \epsilon)|\bar{c}\gamma_\mu \gamma_5 b|B(v)\rangle &= i\sqrt{m_B m_{D^*}} [\xi_{A1}(y)(vv' + 1)\epsilon_\mu^* - \xi_{A2}(y)(\epsilon^* v)v_\mu \\ &\quad - \xi_{A2}(y)(\epsilon^* v)v'_\mu],\end{aligned}$$

where we have defined $y = vv'$. In the heavy-quark limit, these form factors are related to the Isgur–Wise function ξ by

$$\xi_i(y) = \xi(y) \text{ for } i = +, V, A1, A3, \quad \xi_i(y) = 0 \text{ for } i = -, A2. \quad (4)$$

The normalization statement (2) may be used to perform a model-independent determination of V_{cb} from semileptonic heavy-to-heavy decays by extrapolating the lepton spectrum to the kinematic endpoint $v = v'$. Using the mode $B \rightarrow D^{(*)}\ell\nu$ one obtains the relation

$$\lim_{v \rightarrow v'} \frac{1}{\sqrt{(vv')^2 - 1}} \frac{d\Gamma}{d(vv')} = \frac{G_F^2}{4\pi^3} |V_{cb}|^2 (m_B - m_{D^*})^2 m_{D^*}^3 |\xi_{A1}(1)|^2. \quad (5)$$

In the heavy-quark limit the form factor ξ_{A1} reduces to the Isgur–Wise function and is unity at the non-recoil point; aside from $|V_{cb}|$ everything in the right-hand side is known.

However, there are corrections to the normalization of the form factor ξ_{A1} at zero recoil, which may be addressed using HQET. A complete discussion of the corrections may be found in the review article by Neubert [2], including reference to the original papers. Here we only state the final result

$$\xi_{A1}(1) = x^{6/25} \left[1 + 1.561 \frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi} - \frac{8\alpha_s(m_c)}{3\pi} \right] \quad (6)$$

$$+ z \left\{ \frac{25}{54} - \frac{14}{27} x^{-9/25} + \frac{1}{18} x^{-12/25} + \frac{8}{25} \ln x \right\} - \frac{\alpha_s(\bar{m})}{\pi} \frac{z^2}{1-z} \ln z \quad (7)$$

$$+ \delta_{m^2} \quad (8)$$

where we use the abbreviations

$$x = \frac{\alpha_s(m_c)}{\alpha_s(m_b)}, \quad z = \frac{m_c}{m_b}, \quad m_c < \bar{m} < m_b.$$

Contributions (6) and (7) originate from QCD radiative corrections. Since for scales above m_b and below m_c the currents are conserved, logarithmic corrections from running will only be induced from scales between m_b and m_c . The contribution (6) is the correction in leading and next-to-leading logarithmic approximation; including the non-logarithmic one-loop contributions.

Corrections of order $z = m_c/m_b$ may only be induced by QCD radiative corrections. The first term in (7) is obtained by keeping the contributions of order $1/m_b$ in the matching at the scale m_b , performing the renormalization group running for these operators and matching at the scale m_c . In this way a resummation of logarithmic terms of the form $\alpha_s z \ln z$ is achieved. However, since $z \sim 0.3$ is not particularly small, one may as well choose to perform the matching from full QCD to the effective theory in only one step, neglecting the running between m_b and m_c . In this way one may retain the full dependence on z , at the price, however, of a scale ambiguity ($m_c < \bar{m} < m_b$) in the choice of α_s . The second correction in (7) is obtained by this procedure, subtracting the linear term in z , which is already contained in the first term.

Finally there are recoil corrections to the normalization of ξ_{A1} . It has been shown [5] that the terms linear in Λ/m_c and Λ/m_b have to vanish due to heavy-quark symmetry. Thus the first non-vanishing recoil corrections are of order $(\Lambda/m_c)^2$, $(\Lambda/m_b)^2$ and $\Lambda^2/(m_b m_c)$. These contributions may only be estimated, since they need an input beyond heavy-quark effective theory. There are various estimates for these corrections, which are compatible with one another

$$\delta_{m^2} = -2\% \dots - 3\% [6], \quad \delta_{m^2} = 0 \dots - 5\% [7], \quad \delta_{m^2} = 0\% \dots - 8\% [8]$$

Adding all the corrections to the normalization, one obtains for the form factor ξ_{A1}

$$\xi_{A1}(1) = 0.96 \pm 0.03, \tag{9}$$

where the error quoted is due to the uncertainty of the $1/m^2$ corrections.

This result has been used to extract V_{cb} from data. In Fig. 1 the latest data [9] are shown. Three different forms of the Isgur–Wise function have

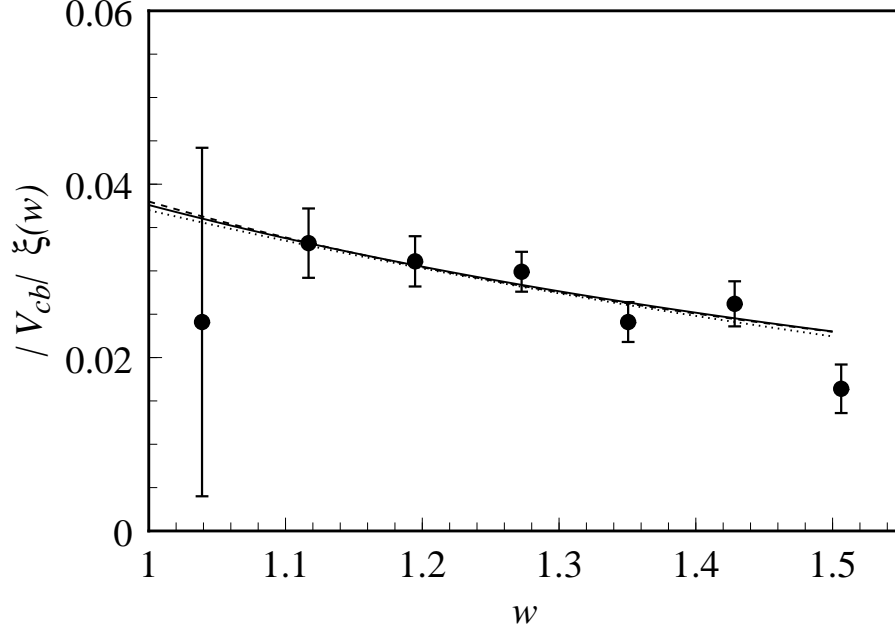


Figure 1: Latest data [9] for the product $|V_{cb}|\xi(vv')$ as a function of $\omega = vv'$. Figure taken from [11].

been used for the extrapolation; however, the difference between the curves is almost invisible. From this fit one obtains

$$|V_{cb}|\left(\frac{\tau_B}{1.5\text{ps}}\right)^{1/2} = 0.039 \pm 0.006. \quad (10)$$

This value of $|V_{cb}|$ is compatible with the value obtained in other ways, e.g. from inclusive decays [10].

3 Inclusive Decays of Heavy Mesons

In order to obtain a $1/m_Q$ expansion for inclusive decay rates one uses an operator product expansion [4] as in deep inelastic scattering. Starting from

the effective Hamiltonian

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{bq} (\bar{b} \gamma_\mu (1 - \gamma_5) q) (\bar{\ell} \gamma^\mu (1 - \gamma_5) \nu), \quad (11)$$

one writes the total inclusive rate as

$$\Gamma = \frac{1}{2m_B} \sum_X (2\pi)^4 \delta^4(P_B - q - P_X) |\langle X_s | H_{eff} | B(v) \rangle|^2. \quad (12)$$

The matrix element appearing in (12) contains a large scale, the mass of the b quark. To make the dependence on this scale explicit, one redefines the b quark field by removing a phase factor corresponding to an on-shell b quark moving with the velocity of the meson

$$b(x) = \exp(-im_b vx) Q_v(x). \quad (13)$$

Inserting this in (12),

$$\Gamma = \frac{1}{2m_B} \int d^4x \exp(im_b vx) \langle B(v) | \widetilde{H}_{eff}(x) \widetilde{H}_{eff}^\dagger(0) | B(v) \rangle, \quad (14)$$

where the tilde denotes the effective Hamiltonian with b replaced by Q_v . Once the phase factor is extracted from the matrix element, it no longer depends on the large mass, and a short-distance expansion may be performed for the operator product appearing in the matrix element. The relevant momentum is $m_b v$ and the short-distance expansion has the form

$$\int d^4x e^{im_b vx} T [\widetilde{H}_{eff}(x) \widetilde{H}_{eff}^\dagger(0)] = \sum_{n=0}^{\infty} \left(\frac{1}{2m_b} \right)^n C_{n+3}(\mu) \mathcal{O}_{n+3}(\mu),$$

where \mathcal{O}_n are operators of dimension n , renormalized at scale μ , and C_n are the corresponding Wilson coefficients.

The lowest-order term of the operator product expansion is the dimension-three operator $\mathcal{O}_3 = \bar{Q}_v Q_v$, and its forward matrix element is normalized because of heavy-quark symmetries. Evaluating this contribution yields the free-quark decay rate.

All dimension-four operators are proportional to the equations of motion $\mathcal{O}_4 \propto \bar{Q}_v (ivD) Q_v$, and the first non-trivial contribution comes from

dimension-five operators and are of order of $1/m_b^2$. For mesonic decays there are two matrix elements of dimension-five operators:

$$\langle B(v) | \bar{h}_v^{(b)} (iD)^2 h_v^{(b)} | B(v) \rangle = 2m_b \lambda_1 \quad \text{and} \quad \langle B(v) | \bar{h}_v^{(b)} i g \sigma_{\mu\nu} G^{\mu\nu} h_v^{(b)} | B(v) \rangle = 12m_b \lambda_2,$$

where $4\lambda_2 = m_{B^*}^2 - m_B^2$. The parameter λ_1 is not as easily accessible, but the expectations from QCD sum rules are $\lambda_1 = -0.52 \pm 0.12 \text{ GeV}^2$ [12].

In terms of these two parameters the non-perturbative corrections to the inclusive decay $B \rightarrow X_u \ell \nu$ is given by the expression

$$\Gamma(B \rightarrow X_u \ell \nu) = \Gamma_b \left[1 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2} \right], \quad \Gamma_b = \frac{G_F^2 m_b^5}{192\pi^3} |V_{ub}|^2; \quad (15)$$

a similar expression may be obtained for the rate for $B \rightarrow X_c \ell \nu$.

The spectrum of the charged lepton in inclusive decays may be calculated by similar means. The rate is written as a product of the hadronic and leptonic tensor

$$d\Gamma = \frac{G_F^2}{4m_B} |V_{Qq}|^2 W_{\mu\nu} \Lambda^{\mu\nu} d(PS), \quad (16)$$

where $d(PS)$ is the phase-space differential. The operator product expansion along the lines described above is then performed for the two currents appearing in the hadronic tensor. Redefining the heavy-quark fields as in (13) one finds that the momentum transfer variable relevant for the short-distance expansion is $m_b v - q$, where q is the momentum transfer to the leptons.

Again the contribution of the dimension-three operators yields the free-quark decay spectrum, and there are no contributions from dimension-four operators. The $1/m_b^2$ corrections are parametrized in terms of λ_1 and λ_2 and the result for the lepton spectrum in $B \rightarrow X_u \ell \nu$ is given by

$$\begin{aligned} \frac{1}{\Gamma_b} \frac{d\Gamma}{dy} &= 2y^2(3-2y) + \frac{10y^2}{3} \frac{\lambda_1}{m_b^2} + 2y(6+5y) \frac{\lambda_2}{m_b^2} \\ &\quad - \frac{\lambda_1 + 33\lambda_2}{3m_b^2} \delta(1-y) - \frac{\lambda_1}{3m_b^2} \delta'(1-y), \end{aligned} \quad (17)$$

where $y = 2E_\ell/m_b$ is the rescaled energy of the charged lepton.

Aside from regular terms, the result also exhibits δ -function singularities at the endpoint, indicating that higher terms in the operator product expansion become important; here $(m_b v - q)^2$ becomes small.

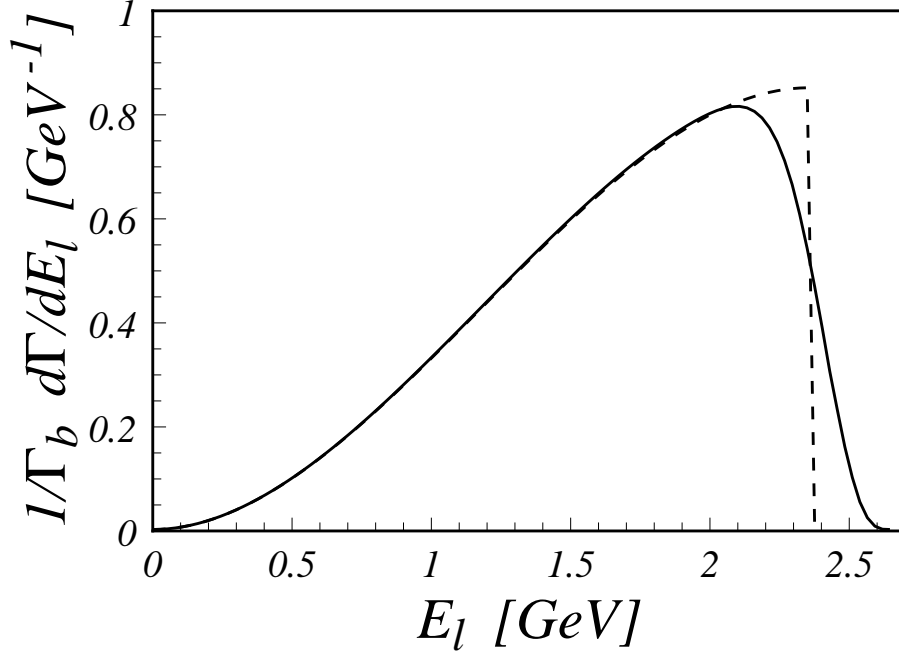


Figure 2: Charged-lepton spectrum in $B \rightarrow X_u \ell \bar{\nu}$ decays. The solid line is (19) with the ansatz (20), the dashed line shows the prediction of the free-quark decay model.

It has been shown [13] that the most singular terms of the short-distance expansion may be resummed into a structure function, analogous to a parton distribution function known from deep inelastic scattering. It is defined formally as

$$f(k_+) = \frac{1}{2m_B} \langle B(v) | \bar{h}_v \delta(k_+ - iD_+) h_v | B(v) \rangle \quad \text{with} \quad k_+ = k_0 + k_3. \quad (18)$$

Using this function, the result for the spectrum becomes

$$\frac{d\Gamma}{dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{12\pi^3} E_\ell^2 (3m_b - 4E_\ell) \int_{2E_\ell - m_b}^{\bar{\Lambda}} dk_+ f(k_+), \quad (19)$$

where $\bar{\Lambda} = m_B - m_b$. The function f is genuinely non-perturbative; in order

to illustrate its effect, we choose a simple one-parameter model

$$f(k_+) = \frac{32}{\pi^2 \bar{\Lambda}} (1-x)^2 \exp \left\{ -\frac{4}{\pi} (1-x)^2 \right\} \Theta(1-x); \quad x = \frac{k_+}{\bar{\Lambda}}, \quad (20)$$

and Fig. 2 shows the resulting spectrum for $\bar{\Lambda} = 570$ GeV. Due to non-perturbative effects, the spectrum now extends beyond the parton model endpoint $m_b/2$ up to the “physical” endpoint $m_B/2$. However, before these results may be confronted with data, the QCD radiative corrections need to be taken into account. First results have been published [14], but the issue is still in a state of flux.

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